Double Angle Identities

These notes are intended as a companion to section 7.6 (p. 652 – 657) in your workbook. You should also read the section for more complete explanations and additional examples.

Double Angle Identities

Use $\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta$ to prove the identity below.

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta$$

This is known as a **double-angle identity**. There are similar identities involving cosine and tangent:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Note: $\cos 2\theta$ can also be written as

$$\cos 2\theta = 2\cos^2\theta - 1 \qquad \cos 2\theta = 1-2\sin^2\theta$$

Example 1 (sidebar p. 654)

Given angle θ is in standard position with its terminal arm in Quadrant 4 and $\cos\theta = \frac{2}{5}$, determine the exact value of each trigonometric ratio.

a) $\sin 2\theta$

b) $\cos 2\theta$

Example 2 (sidebar p. 655)
Write each expression as a single trigonometric ratio, then evaluate where possible.

a) $\cos^2\left(\frac{\pi}{4}\right)-\sin^2\left(\frac{\pi}{4}\right)$

b) $\dfrac{2\tan\frac{\pi}{6}}{\tan^2\left(\frac{\pi}{6}\right)-1}$

c) $2\sin\frac{\pi}{12}\cdot\cos\frac{\pi}{12}$

Example 3 (sidebar p. 656)
Prove each identity.

a) $\cot\theta = \dfrac{\cos 2\theta + 1}{\sin 2\theta}$

b) $\cot\theta \cdot \csc 2\theta = \dfrac{1}{2\sin^2\theta}$

Example 4 (sidebar p. 657)

Solve the equation $\frac{1}{2}\sin 2x - \cos^2 x = 0$ over the domain $0 \le x < 2\pi$.

Homework: #4 – 6, 9 – 15, 17 in the exercises (p. 658 – 665). Answers on p. 666.